HW #5 MMAE 530

* **Problem**: Stresses in axisymmetric plane problems are given by

$$\sigma_r = \frac{1}{r}\frac{d\phi}{dr} = \frac{c}{r^2} + A(1+2\ln r) + 2B$$

$$\sigma_\theta = \frac{d^2\phi}{dr^2} = -\frac{c}{r^2} + A(3+2\ln r) + 2B$$

$$\tau_{r\theta} = 0$$

Uniqueness of displacements requires that $\underline{A=0}$. This can be shown by substituting Hooke's Law in the compatibility equation;

$$\varepsilon_r - \varepsilon_\theta = r\frac{d\varepsilon_\theta}{dr}$$

together with the stress components given above, this yields $\underline{A=0}$. PROOVE THIS.

* **Solution**:

Hooke's Law:

$$\left.\begin{aligned}\varepsilon_r &= \frac{1}{E}\left[\sigma_r - \nu(\sigma_\theta + \sigma_z)\right]\\ \varepsilon_\theta &= \frac{1}{E}\left[\sigma_\theta - \nu(\sigma_r + \sigma_z)\right]\end{aligned}\right\} \Rightarrow \begin{aligned}\varepsilon_r - \varepsilon_\theta &= \frac{1}{E}\left[(\sigma_r - \sigma_\theta) - \nu(\sigma_\theta - \sigma_r)\right]\\ &= \frac{1+\nu}{E}(\sigma_r - \sigma_\theta)\end{aligned}$$

$$\Rightarrow \varepsilon_r - \varepsilon_\theta = \frac{2(1+\nu)}{E}\left[\frac{c}{r^2} - A\right] \quad \text{------ (1)}$$

- FOR PLANE STRESS $\longrightarrow$ $\varepsilon_\theta = \frac{1}{E}(\sigma_\theta - \nu\sigma_r)$

$$\Rightarrow r\frac{d\varepsilon_\theta}{dr} = \frac{r}{E}\left(\frac{d\sigma_\theta}{dr} - \nu\frac{d\sigma_r}{dr}\right)$$

$$r\frac{d\varepsilon_\theta}{dr} = \frac{2}{E}\left[\frac{(1+\nu)}{r^2}c + (1-\nu)A\right] \quad \text{----- (2a)}$$

• FOR PLANE STRAIN $\longrightarrow$ $\varepsilon_\theta = \frac{1+\nu}{E}\left[(1-\nu)\sigma_\theta - \nu\sigma_r\right]$

$$\Rightarrow \quad r\frac{d\varepsilon_\theta}{dr} = \frac{2}{E}\left[\frac{(1+\nu)}{r^2}C + (1+\nu)(1-2\nu)A\right] \quad \text{------} (2b)$$

Compatibility requires:

$$\underbrace{\varepsilon_r - \varepsilon_\theta}_{\text{Eqn.}(1)} = \underbrace{r\frac{d\varepsilon_\theta}{dr}}_{\text{Eqn.}(2a) \text{ or Eqn.}(2b)}$$

Eqn. (2a) $\uparrow$ for pl-σ or Eqn. (2b) $\uparrow$ for pl.ε

$$\Rightarrow \quad \frac{\cancel{2}(1+\nu)}{\cancel{E}}\left[\frac{C}{r^2} - A\right] = \frac{\cancel{2}}{\cancel{E}}\left[\frac{(1+\nu)}{r^2}C + (1-\nu)A\right] \quad \text{for pl-}\sigma$$

OR

$$= \frac{\cancel{2}}{\cancel{E}}\left[\frac{(1+\nu)}{r^2}C + (1+\nu)(1-2\nu)A\right] \quad \text{for pl-}\varepsilon$$

$$\Rightarrow \quad \frac{C}{r^2} - A = \frac{C}{r^2} + \frac{(1-\nu)}{(1+\nu)}A$$

OR

$$= \frac{C}{r^2} + (1-2\nu)A$$

it is obvious from the comparison of left hand side with the right hand side that <u>A has to be zero</u> in order to satisfy compatibility condition for both pl-σ and pl-ε.