Directions: Carefully write the solutions to all the problems showing all steps and work. Do not write answers in decimal form - use whole numbers or fractions.

1. Find $A^{-1}$, if possible, using the augmented matrix and transforming $[A|I] \rightarrow [I|A^{-1}]$ where $A = \begin{pmatrix} 1 & 2 & 2 \\ 2 & 3 & 3 \\ 3 & 4 & 5 \end{pmatrix}$. Verify that you have the correct inverse by showing $A^{-1}A = I$.

2. Transform $\begin{pmatrix} 1 & 2 & 2 & 3 \\ 2 & 5 & 4 & 8 \\ -1 & -3 & -2 & -5 \\ 0 & 2 & 0 & 4 \end{pmatrix}$ into row-echelon form. Show all steps and indicate what operations were used.

3. Find the solution set of $A\vec{x} = \vec{b}$ if the augmented matrix in row-echelon form is $\begin{pmatrix} 1 & -2 & 4 & 1 \\ 0 & 1 & -1 & 3 \end{pmatrix}$. Write the solution as a particular solution plus a solution of the homogeneous equation $A\vec{x} = 0$.

4. Find the null space of $A$, i.e. find the solutions of $A\vec{x} = 0$, if the row-echelon form of $A$ is $A = \begin{pmatrix} 1 & 4 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{pmatrix}$.

5. Find the solution set of $A\vec{x} = \vec{b}$ if the augmented matrix in row-echelon form is $\begin{pmatrix} 1 & 3 & 2 & -2 \\ 0 & 0 & 1 & 4 \\ 0 & 0 & 0 & 1 \end{pmatrix}$.

6. Consider the system

\[\begin{align*}
x_1 + 3x_2 + x_3 + x_4 &= 3 \\
2x_1 - 2x_2 + x_3 + 2x_4 &= 8 \\
x_1 - 5x_2 + x_4 &= 5
\end{align*}\]

(a) Is the system overdetermined or underdetermined?
(b) Find the solution using the Gaussian Elimination on the augmented matrix. **CHECK**
(c) Write the solution as the sum a particular solution plus solutions of the homogeneous equation.
(d) What is the dimension of the null space? In other words, how many nonzero vectors are in the null space.