Directions: Carefully write the solutions to all the problems showing all steps and work. Please submit a neat document. Do not write answers in decimal form - use whole numbers or fractions.

Misc.

1. Use the quadratic formula to find the roots of \( z^2 + (1 - i)z - i = 0 \). Explicitly check that the roots satisfy the equation.

2. Find the steady-state solution \( y(t) \) that satisfies \( y''(t) + 4y(t) = 10 \cos(3t + \pi) \). You are to convert to a new problem with the right side of the ODE containing a complex exponential. Using the class notes, guess a solution of the form \( y = Ae^{3ti} \) and find \( A \). Finally, find the real part of your solution which is the solution of the original problem. Your solution must contain the term \( (3t + \pi) \).

Section 1.7

3. Express \( e^z \) at \( z = -1 + \pi i / 4 \) in \( a + bi \) form.

4. Express \( f(z) = e^{2\overline{z}} \) in \( u + iv \) form.

5. Express \( \ln(-2 + 2i) \) in \( a + bi \) form.

6. Solve \( e^{z-1} = -ie^2 \).

7. If \( z_1 = i \) and \( z_2 = -1 + i \), does \( \ln(z_1z_2) = \ln(z_1) + \ln(z_2) \).

8. Express \( \sin(-2i) \) in \( a + bi \) form.

9. Find all values of \( z \) such that \( \cos z = -3i \).

Section 2.1

10. Evaluate \( f(z) = 4z + i\overline{z} - \text{Re}(z) \) at \( z = 4 - 6i \).

11. At what points are the following functions not analytic

   (a) \( \frac{z}{z - 3i} \)

   (b) \( \frac{z^2 - 2iz}{z^2 + 4} \)
12. The function \( f(z) = z^3 \) is analytic for all \( z \).

(a) Show the Cauchy-Riemann equations are satisfied for all \( z \).

(b) Compute \( f'(z) \) using the differentiation rules since \( f \) is only a function of \( z \).

(c) Use the Cauchy-Riemann equations to compute \( f' \) and show the result is the same as in (b).

13. Compute \( f' \) is \( f(z) = \frac{4z^3 - 5z + 1}{2z - 1} \).

14. Use the Cauchy-Riemann equations to show that the following functions are not analytic at any point. Explain your conclusion.

(a) \( f(z) = y + ix \)

(b) \( f(z) = \bar{z}^2 \)

(c) \( f(z) = 2x^2 + y + i(y^2 - x) \)