S1 Suppose the simple function $\phi$ can be written as two different ways: $\phi = \sum_{i=1}^{m} a_i \chi_{A_i} = \sum_{j=1}^{n} b_j \chi_{B_j}$, where $a_i, b_j$’s are nonnegative for all $i$’s and $j$’s. Show that $\sum_{i=1}^{m} a_i \mu(A_i) = \sum_{j=1}^{n} b_j \mu(B_j)$. (Thus, the Lebesgue integral definition is independent of the different representations of the simple function.)

S2 Let $(X, \mathcal{A}, \mu)$ be a measure space and suppose $\mu$ is $\sigma$-finite. Suppose $f$ is integrable. Prove that given any $\epsilon > 0$ there exists $\delta$ such that

$$\int_{A} |f(x)| \mu(dx) < \epsilon$$

whenever $\mu(A) < \delta$. 