1. (5 points)
Multistep methods are usually implemented in ‘predictor-corrector’ form. That is, a preliminary calculation is carried out using an explicit method (e.g. Adams-Bashforth) and the resulting approximate solution at next time level is used in an implicit (e.g. Adams-Moulton) method of the same order to approximate the derivative at the new time level. For example, the ‘Predict-Evaluate-Correct-Evaluate’ (PECE) scheme based on Adams-Bashforth of order 2 and BDF for $s = 2$ can be written as follows.

\begin{align*}
y_{n+2}^* &= y_{n+1} + \frac{h}{2} [3f(t_{n+1}, y_{n+1}) - f(t_n, y_n)] \quad (1) \\
y_{n+2} &= \frac{4}{3}y_{n+1} - \frac{1}{3}y_n + \frac{2h}{3} f(t_{n+2}, y_{n+2}^*) \quad (2)
\end{align*}

Investigate whether the PECE scheme (1)-(2) is A-stable.

2. (5 points)
Find if the fourth-order Lobatto IIIB method (given in Ex. 5.3 of Iserles) is algebraically stable.

3. (20 points, Computer Problem)
Consider the boundary value problem (BVP)

\begin{equation}
y'' = y^3 - yy', \quad 1 \leq t \leq 2, \quad y(1) = \frac{1}{2}, \quad y(2) = \frac{1}{3}.
\end{equation}

(a) Use the shooting method (along with the 4th-order classic Runge-Kutta method and Newton’s method) to solve the BVP with $h = 0.1$. Plot the error of the numerical solution. The analytic solution is $y(t) = 1/(1 + t)$.

(b) Use the finite difference method to solve the BVP with $h = 0.1$. Again, plot the numerical error.

(c) Discuss which numerical solution is more accurate or efficient for this BVP.

4. (5 points, Bonus Computer Problem)
For the previous BVP, could you apply Richardson extrapolation to the finite difference solution to get more accurate numerical result? Plot the numerical error of your extrapolated solution.