3-DIMENSIONAL COORDINATE SYSTEMS, LONGITUDE AND LATITUDE

In the figure:

\( O \) = Center of the Earth.

Point \( P \) on the Earth’s surface has

Latitude = \( \varphi \) and Longitude = \( \lambda \).

In general, we take:

- \(-180^\circ < \lambda \leq +180^\circ\)
- \(-90^\circ \leq \varphi \leq +90^\circ\)

\( \rho = |OP| = 3960 \text{ miles} \).

With respect to the Cartesian coordinate axes: \( x, y \) and \( z \) we have

\[
P = (x_P, y_P, z_P) = (\rho \cos \lambda \cos \varphi, \rho \sin \lambda \cos \varphi, \rho \sin \varphi).
\] (1)

If \( \rho_{OQ} = |OQ| \) = distance from \( Q \) to the center of Earth in the figure above, then

\[
Q = (x_Q, y_Q, z_Q) = (\rho_{OQ} \cos \lambda \cos \varphi, \rho_{OQ} \sin \lambda \cos \varphi, \rho_{OQ} \sin \varphi).
\] (2)

The straight line linear distance \( |P_1 P_2| \) between any two points \( P_1 \) and \( P_2 \) in 3-space may be computed from their Cartesian coordinates.

If \( P_1 = (x_1, y_1, z_1) \) and \( P_2 = (x_2, y_2, z_2) \) then

\[
|P_1 P_2| = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2}.
\] (3)

If \( \theta = \angle POR \) is the angle formed between any two points \( P \) and \( R \) on the surface of the Earth with \( O \), the center of the Earth, then

\[
\theta = \cos^{-1} [\sin \varphi_P \sin \varphi_R + \cos \varphi_P \cos \varphi_R \cos (\lambda_P - \lambda_R)].
\] (4)

In the above formula, assume \( P \) has (Longitude= \( \lambda_P \); Latitude = \( \varphi_P \))

and \( R \) has (Longitude = \( \lambda_R \); Latitude = \( \varphi_R \)).

The geodesic or surface distance, \( SD_{PR} \), between the points \( P \) and \( R \) measured along a great circle of the Earth is

\[
SD_{PR} = \left( \frac{\theta^\circ}{180^\circ} \right) \cdot 3960 \pi \text{ miles}.
\] (5) (where \( \theta^\circ = \angle POR \) in (5))
GEOGRAPHIC FACTS

- The diameter of the Earth at the equator is 7926 miles.
- The diameter of the Earth through the poles is 7900 miles.
- The Earth weighs 598,000,000,000,000,000,000,000 metric tons (598 sextillion).
- Earth orbits around the Sun at 66,000 miles per hour.
- The speed of the Earth spinning on its axis at the equator is about 1250 miles per hour.

- Arctic Circle: 66° 33′ 39″ N = +66.5608°
- Tropic of Cancer: 23° 26′ 21″ N = +23.6058°
- Tropic of Capricorn: 23° 26′ 21″ S = -23.6058°
- Antarctic Circle: 66° 33′ 39″ S = -66.5608°

- **Greenwich, England**
  Latitude = 51.4788, Longitude = -0.0107
  Lat = 51 degrees, 28.7 minutes North
  Long = 0 degrees, 0.6 minutes West

- **Chicago, IL**
  Latitude = 41.8795, Longitude = -87.6243
  Lat = 41 degrees, 52.8 minutes North
  Long = 87 degrees, 37.5 minutes West