

# THE ART OF SPIROLATERALS

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## Abstract

This paper continues an investigation into spirolaterals as geometry to generate artistic forms of unexpected complexity and beauty. This particular phase will concern the two dimensional representations of spirolaterals and its forms based on both same and reversals of turns. Using mathematical and computer-based methods, issues of closure, variation, enumeration, and predictability are discussed. The overriding interest in this research is to develop methods and procedures to investigate geometry for the purpose of inspiration for new architectural and sculptural forms.

## Introduction

Frank C. Odds, a British biochemist, wrote the first apparent description of spirolaterals [4]. Further research uncovered additional description by Abelson [1]. A spirolateral is created by drawing a set of lines; the first at a unit length, then each additional line increasing by one unit length while turning a constant direction. Figure 1 shows the systematic generation of an order 3 spirolateral; one that consists of 3 segments at turns of 90 degrees.

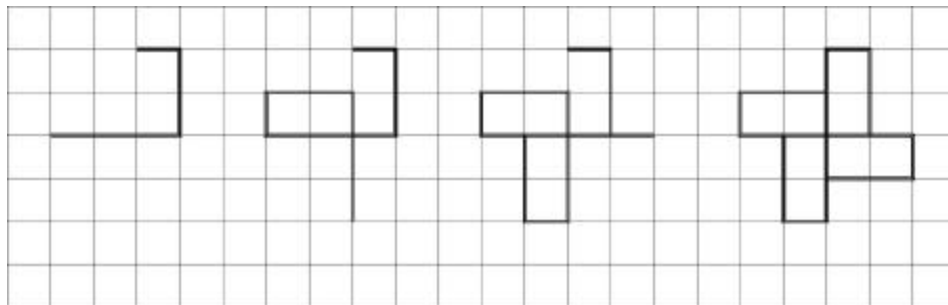


Figure 1: Generation of an order 3 spirolateral

Step 1: turn 90 degrees, draw one unit segment, turn 90 degrees, draw two unit segment, turn 90 degrees, draw three unit segment. Step 2, 3 and 4: repeat Step 1

Odds writes that name *spirolateral* is derived from two roots: *lateral*, referring to a flat surface, and *spiro*, since the original series of spirolaterals was generated from the “square spiral”. Each segment is one unit longer than the one drawn before it, and each segment turns a constant 90 degrees from its predecessor. To complete a closed spirolateral, it is necessary to only to repeat the “square spiral” design, until the starting point is reached.

## Properties of Spirolaterals

From the introductory example, the spirolateral is defined by three basic factors: the turning angle, the number of segments or turns, and the number of repetitions, which create a closed figure.

To demonstrate the closing property of a spirolateral, Figure 2 shows a 90-degree spirolateral from 1 to 10 turns.

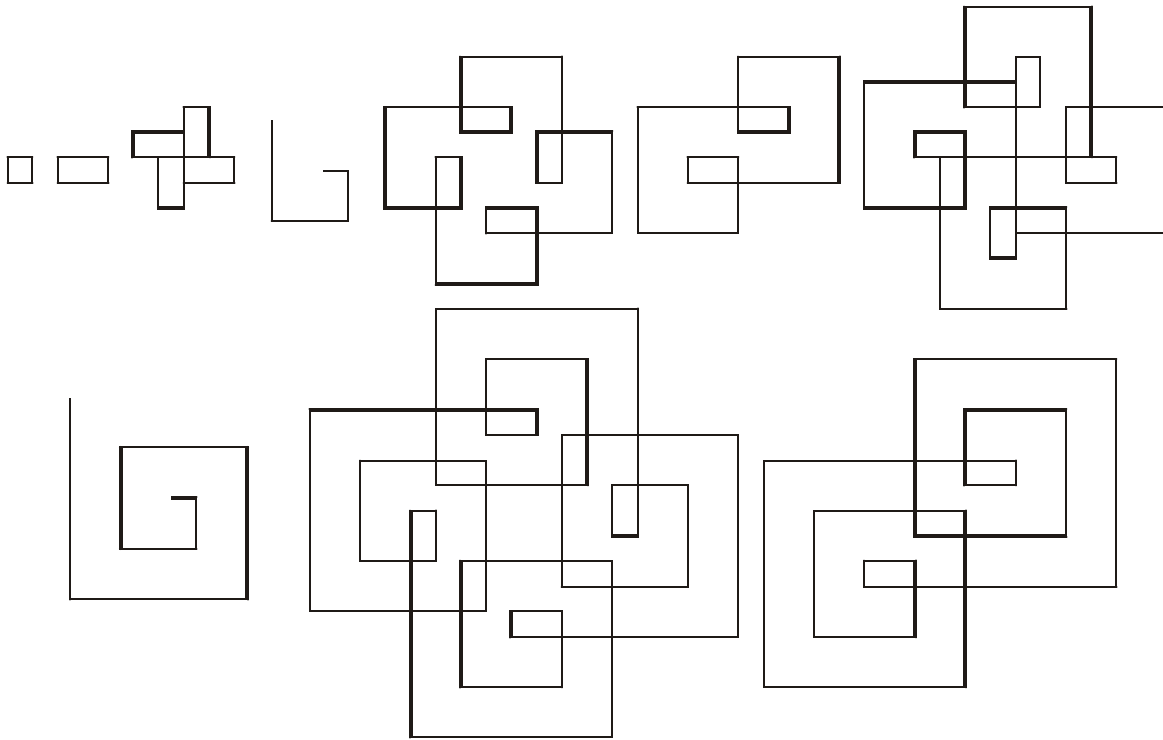


Figure 2: Spirolateral of 90 degrees, from 1 to 10 turns

To predict the closing property of a spirolateral, Krawczyk [3] developed the following relationship. If the total turning angle is modulo 360, then the spirolateral is closed:

$$\text{Total turning angle} = \text{angle of turns} \times \text{number of turns} \times \text{repetitions}$$

The exception is that when only one repetition become 360 degrees, the spirolateral will not be closed, or if repeated further it will just meander outward without ever closing on itself.

Odds developed a notation to describe each of the spirolaterals. The number of turns,  $n$ , referred to as the “order” of the spirolateral, is the base number. The angle of turn is written as a subscript; thus,  $5_{90}$  would define a spirolateral with five turns each through 90 degrees.

Odds suspected that any angle that is an exact divisor of 180 degrees would generate a spirolateral. Gardner [2] also agreed with Odds without investigating any other possible angles. Figure 3 demonstrates closed spirolaterals based on an angle of  $180/n$ .

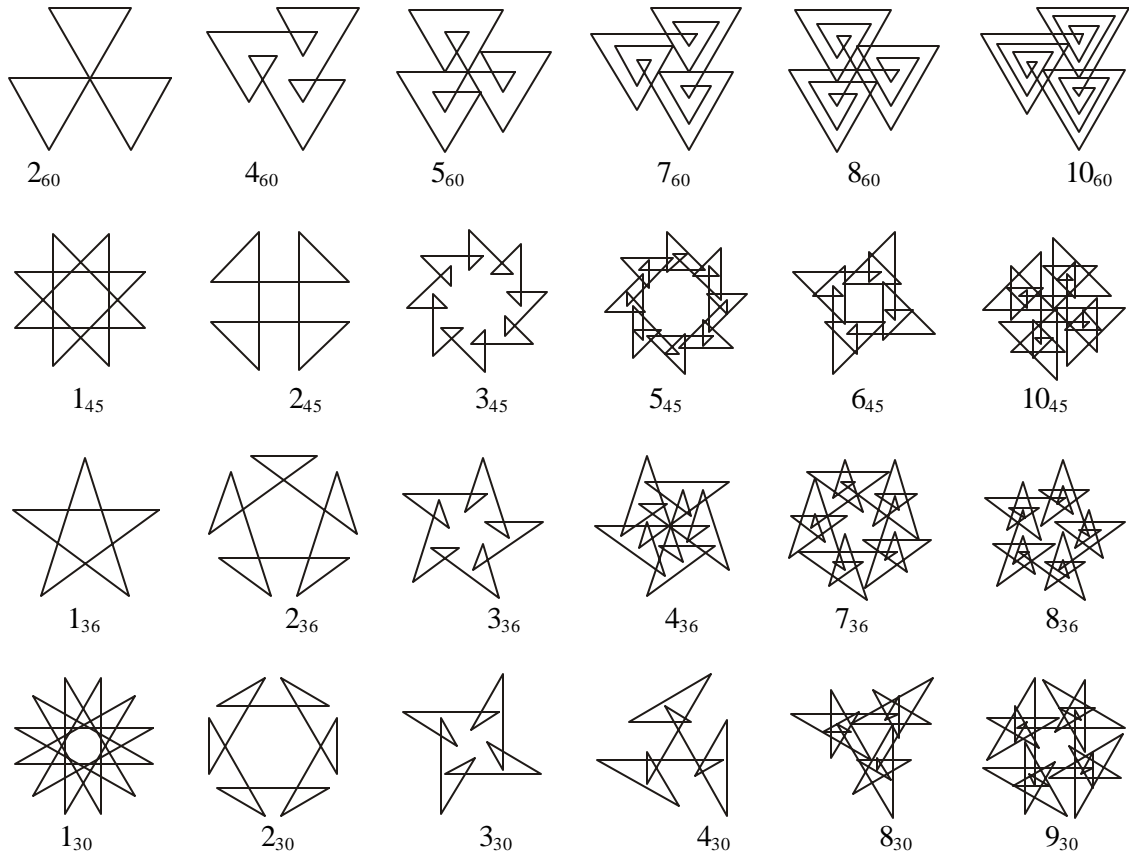


Figure 3: Spirolaterals based on an angle  $180/n$

Another computer program was written to enumerate other spirolaterals. The enumeration included all the angles from 1 to 180, incremented by 1 degree, and number of turns from 1 to 10 were attempted to generate a closed spirolateral, with a maximum of 10 repeats. From 1800 possible spirolaterals, 561 were closed by the 360 degree rule; of the 561, 113 were not actually closed. Of that gallery of 448 images, Figures 4a and 4b exhibit a representative selection.

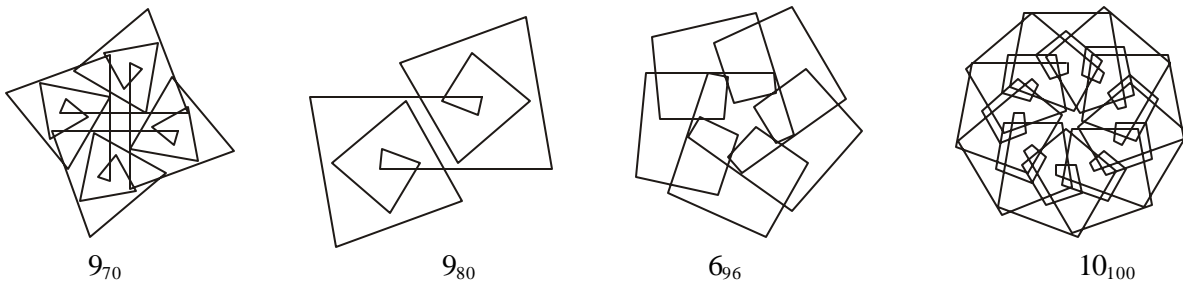


Figure 4a: Spirolaterals based on angles not  $180/n$

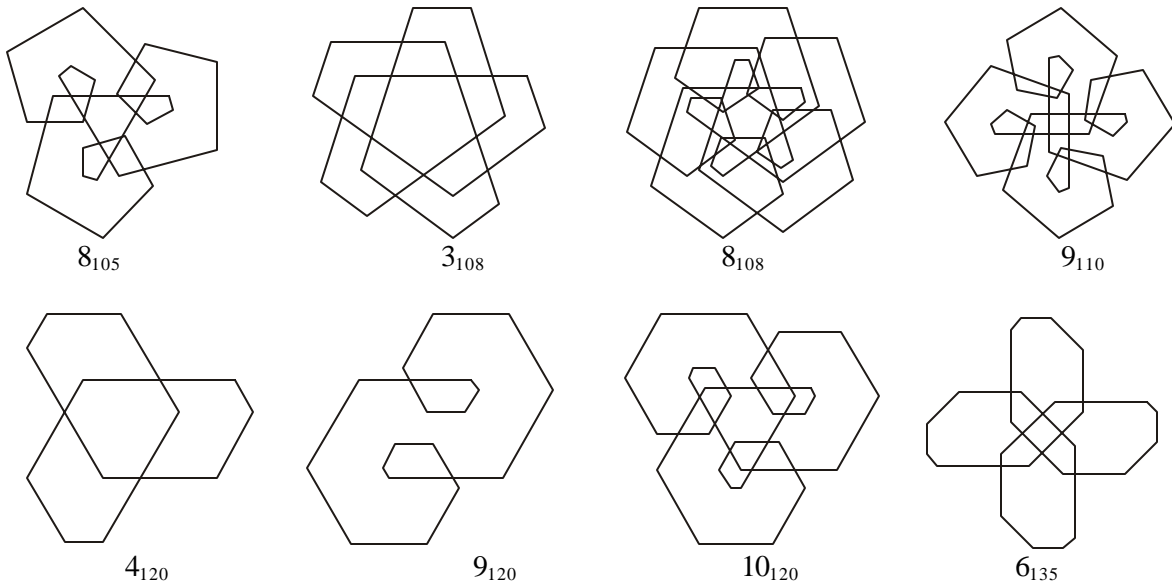


Figure 4b: Spirolaterals based on angles not  $180/n$

## Reversing Turns in Spirolaterals

Odds, in further exploring the possible configurations for spirolaterals, introduced the concept that not all the turns need to be in the same direction. For any series of segments, some of the turns can be to the right and others can be to the left. The notation he suggests is:  $7_{90}^{4,6}$  for a spirolateral of 7 turns at 90 degrees in which the 4<sup>th</sup> and 6<sup>th</sup> turns are in the opposite direction. Figure 5a and 5b show a series of reverse turn spirolaterals based on the angle of 90 degrees.

For a specific order  $n$  there are  $2^n$  possible turns that generate spirolaterals, half of those are mirror images; so  $2^{n-1}$  unique figures are possible in each order. From the analysis of the closure relationship previously described it has been found that spirolaterals, which close for all turns in the same direction, will also close for any reversed turns given the same number of turns and repetitions.

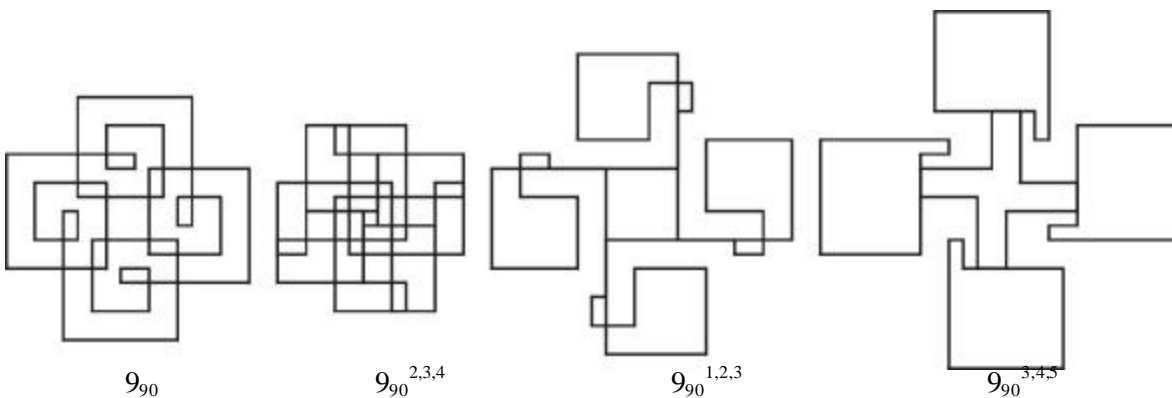


Figure 5a: Spirolaterals with reverse turns

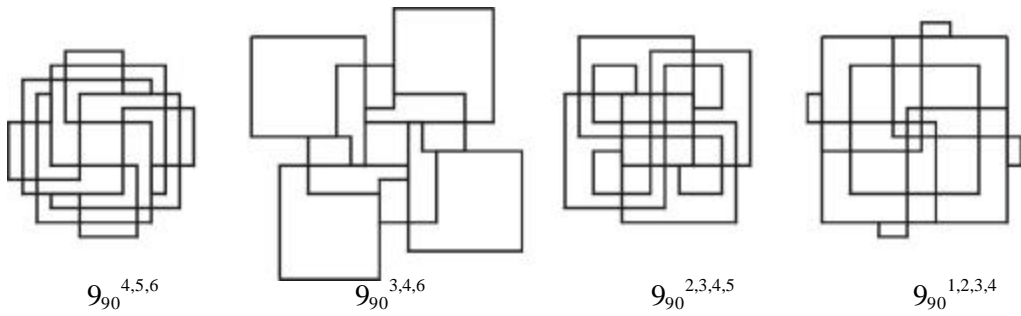


Figure 5b: Spirolaterals with reverse turns

A mathematical relationship was never developed by Odds to account for the unclosed spirolaterals, nor was there one found for the prediction of closing reverse turned spirolaterals. Abelson suggests a method to generate unexpectedly closed spirolaterals by enumeration, but not in a predictive mathematical form.

To be able to investigate the variety of forms generated by reversals, the enumeration method suggested by Abelson was developed. After examination of a few sets of spirolateral reversals, exclusion was predicted by the following methods:

1. Only generate half of the possible reversals to eliminate symmetrical forms
2. If the sum of the left turn angles minus the sum of the right turn angles is equal to zero, then the spirolateral is symmetrical to the original one with all one directional turns
3. If the sum of the left turn angles is equal to the sum of the right turn angles then the spirolateral is open, not closed

Using these rules, a unique set of spirolaterals can be generated. Figure 6 displays the 22 unique reversals of the  $6_{45}$  spirolateral.

Another example is given in Figure 7 of a  $7_{60}$  spirolateral.

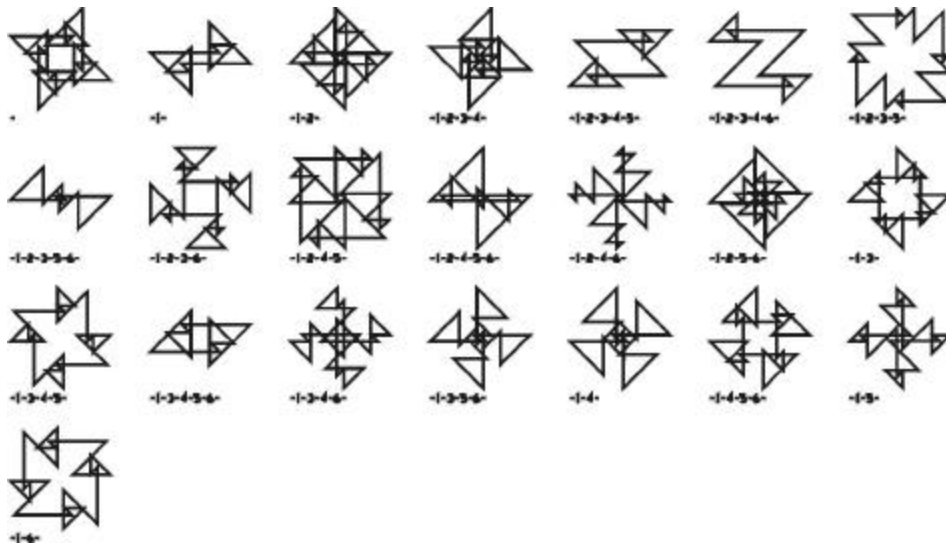


Figure 6:  $6_{45}$  Spirolateral with only unique reverse turns

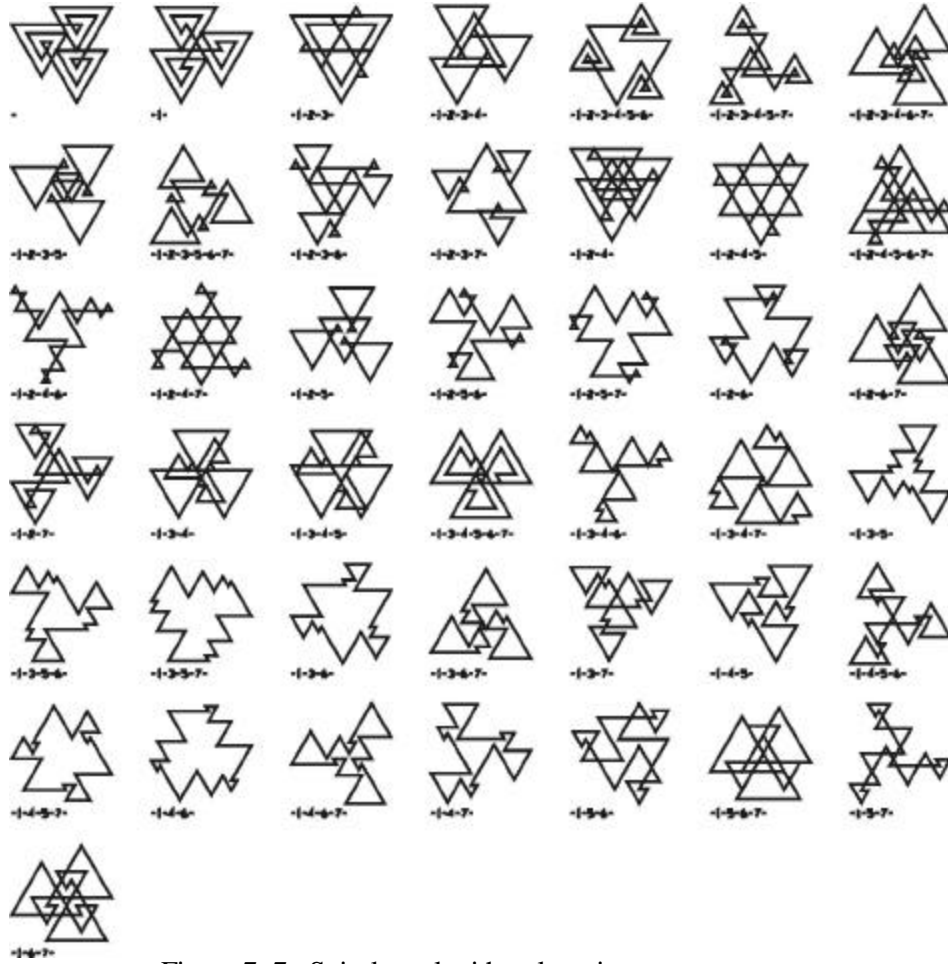


Figure 7:  $7_{60}$  Spirolateral with only unique reverse turns

## Spirolaterals as Form Generators

Spirolaterals can be used explicitly to generate designs. Individual spirolaterals can be replicated into patterns or used singly for many two-dimensional designs in a variety of media. The spirolateral can also be used as an axis or spine for other design elements. For example, the vertices that compose a spirolateral can be used as control points for circular arcs or splines. The interpretation of the line segment can be varied to include unit arcs and zigzags. Figure 8a and 8b demonstrate these concepts..

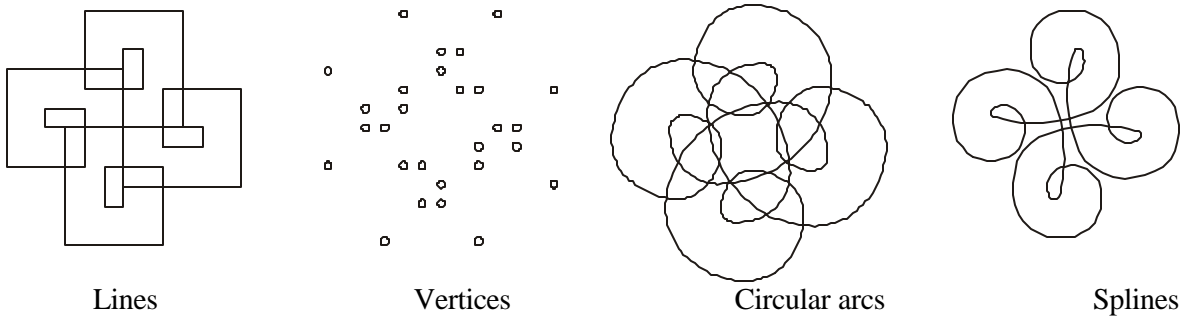


Figure 8a: Designs based on the  $7_{90}$  spirolateral

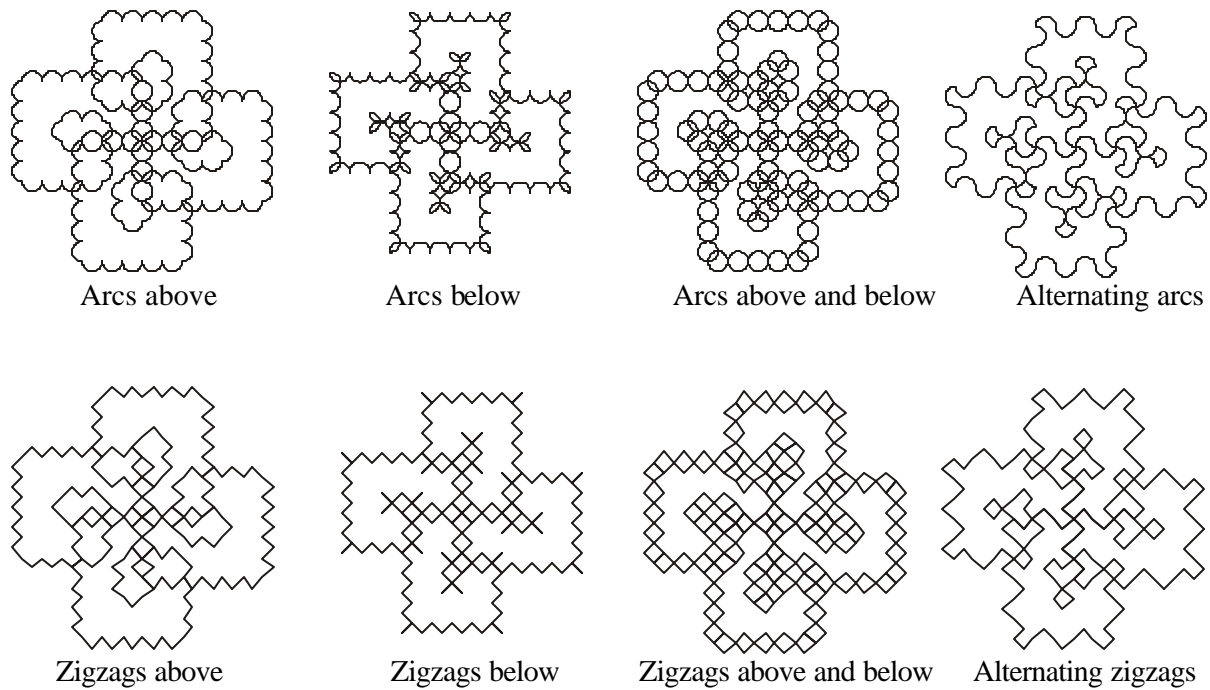


Figure 8b: Designs based on the  $7_{90}$  spiroilateral

## Spirolaterals as Artwork

A number of different attempts have been made to use the forms generated by the spirolaterals in two-dimensions to create actual artwork. Having the design generated within a CAD package allows for the simple drawing of it using a variety of line thickness; no different than drafting the same manually. No attempt has been made to replicate an individual form for the use as a pattern in fabric design or in a similar application. That possibility could still be investigated. A second version was tried using a continuous piece on wire, bent according to a diagram of the design. Somewhat more successful, it gave the design a physical depth but due to overall scale and some of the turning angles, the angles were difficult to reproduce or the smallest length became too small to handle well. In addition, the spiraling and overlapping nature of the designs caused a layering of the wire that made it difficult to work with on complex forms. A third version used a continuous strip of putty on glass to replicate the design. This gave results that were somewhat more interesting. The softer material solved the problem with the turns and overlapping; and the "three-dimensionality" added to the drawing quality of the work. Work continues using this method.

Since the underlying concept is to have these design be able to be generated by anyone and then produced into some type of physical form, a series of programs have been written to generate spirolaterals. The first ones written in Microsoft QuickBASIC for computational inquiry and AutoCAD's AutoLISP for visualization. The first version simply used a single line to represent each spiroilateral. Visually it was very weak. A line thickness was added. The forms turned from simple line drawings to iconic symbols. As the line thickness was increased, the open areas between the lines began to close and a totally different form emerged. Once the basics were understood a JAVA version as written to display the spirolaterals on the web. Figure 9 displays the JAVA version. This version can be found at: [www.iit.edu/~krawczyk/start.htm](http://www.iit.edu/~krawczyk/start.htm).

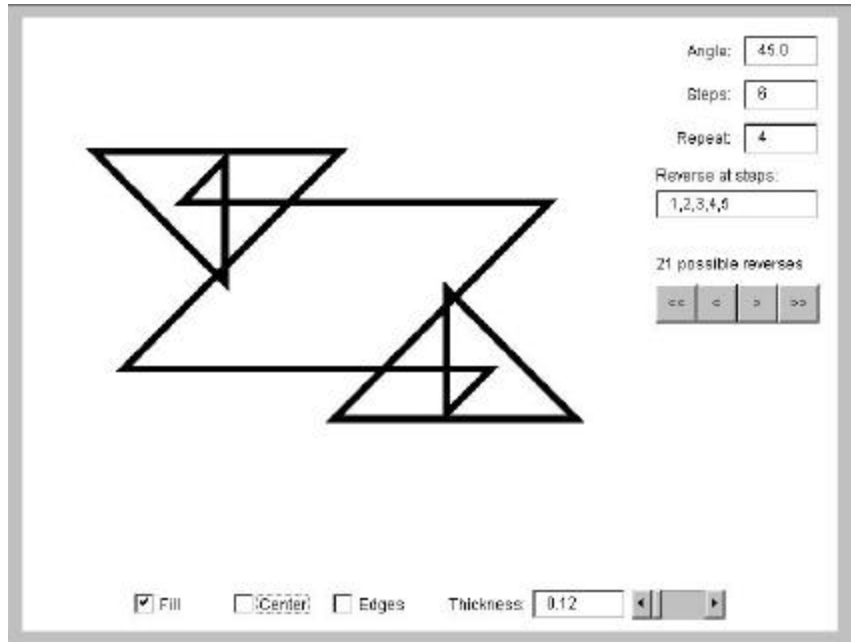
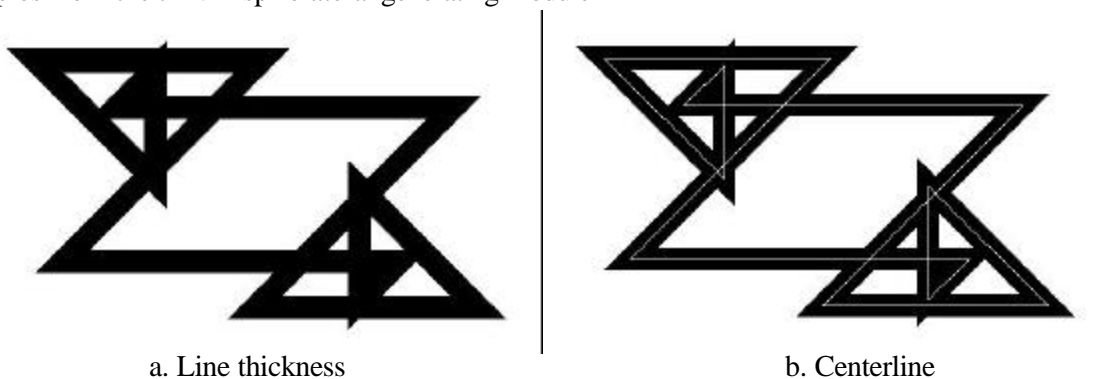


Figure 9: JAVA program to generate spiroilaterals

The current web version includes two JAVA modules. The first has a selected group of spiroilaterals that generate their reversals as a continuous display. Each is created line-by-line, erased, and then another is generated. The second module used the same set of spiroilaterals, but enables the viewer to modify the line thickness and browse through each of the reversals, Figure 10a.

In addition to displaying the line thickness, options are included to add the centerline and edge lines or both for each. Figure 10b, c, and d display these variations. These options displayed the internal geometry of the spiroilateral that was somewhat lost to the line thickness. Figure 11 display selected examples from the JAVA spiroilateral-generating module

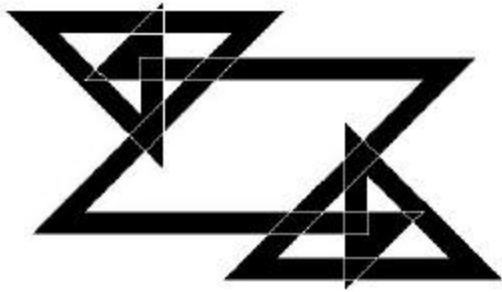


a. Line thickness

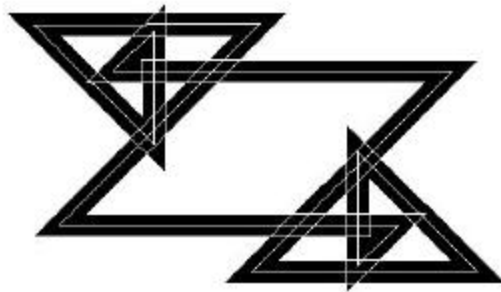
b. Centerline

Figure 10: Variation of thickness and lines





c. Edge lines



d. Center and edge lines

Figure 10: Variation of thickness and lines

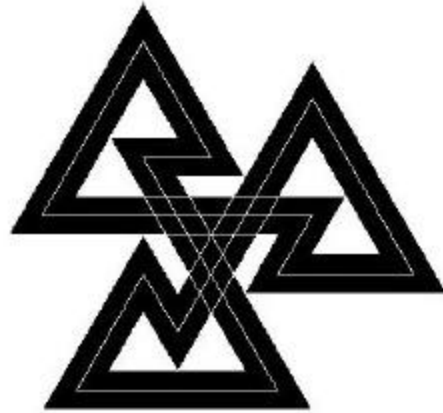
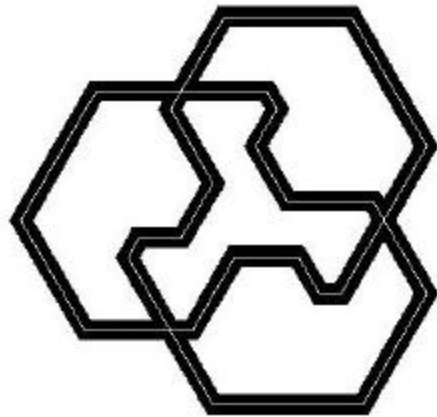


Figure 11: Selected examples

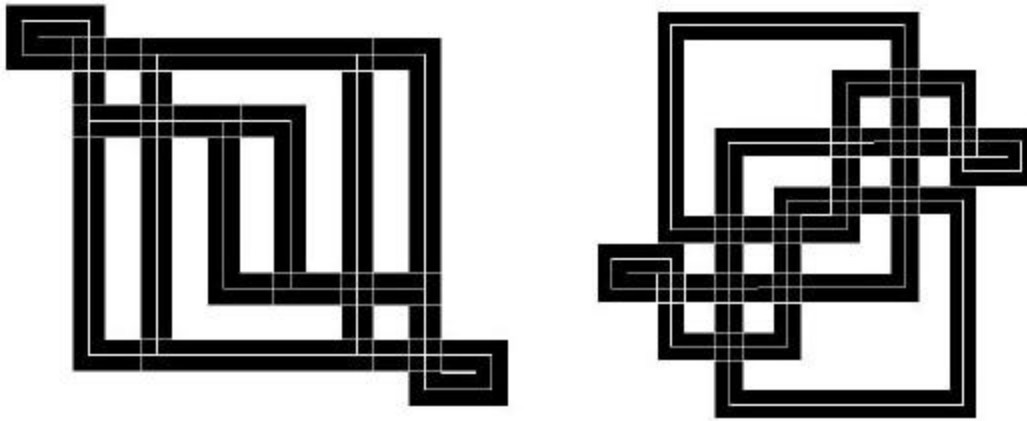


Figure 11: Selected examples

## Web Based Artwork

The development of a web-based version of generating spirolaterals advances some very interesting concepts. The first is the distribution of an artistic idea is readily made with very little cost to the artist and it offers the potential for a very large audience. Another is the ability for viewers to actually investigate an idea beyond what was originally developed. Depending on the time spent and the individual interest of the viewer, the viewer is able to see particular instances that the artist never encountered.

The most intriguing aspect is the potential to deliver completed artwork with such a method. One extension to these JAVA modules is to have one ask the viewer for their email address and then email it with the parameters of a selected spirolateral to the artist. The Artist can then offer the viewer a variety of options of size, mounting, and cost for a final version of the design. The artist would be responsible for the environment that the artwork would exist. The viewer could then discover his or her own vision of the artwork.

The next phase of this research is to review methods to create full three-dimensional sculptures or designs that begin to suggest architectural forms. From the preliminary investigation of these forms, there seems to exist a great variety that has that potential.

Even with the possibility of enumerating a finite number of forms based on a selected angle scheme, when you include turn reversal, there still exist a great number of unexpected designs. The unpredictable, under controlled conditions, is what makes the spirolateral of continuing interest.

## References

- [1] Abelson, Harold, diSessa, Andera, 1968, *Turtle Geometry*, MIT Press, pp.37-39, 120-122
- [2] Gardner, Martin, 1986, *Knotted Doughnuts and Other Mathematical Entertainments*, W. H. Freeman and Company, pp. 205-208
- [3] Krawczyk, Robert, 1999, "Spirolaterals, Complexity from Simplicity", in *International Society of Arts, Mathematics and Architecture 99*, edited by N. Friedman and J. Barrallo, The University of the Basque Country, pp. 293-299
- [4] Odds, Frank, "Spirolaterals", *Mathematics Teacher*, February 1973, pp.121-124