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Wireless Link Scheduling under Physical Interference Model

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***Abstract*—Link scheduling is a fundamental problem in multihop wireless networks because the capacities of the communication links in multihop wireless networks, rather than being fixed, vary with the underlying link schedule subject to the wireless interference constraint. The majority of algorithmic works on link scheduling in multihop wireless networks assume binary interference models such as the 802.11 interference model and the protocol interference model, which often put severe restrictions on interference constraints for practical applicability of the link schedules. On the other hand, while the physical interference model is much more realistic, the link scheduling problem under physical interference model is notoriously hard to resolve and been studied only recently by a few works. This paper conducts a full-scale algorithmic study of link scheduling for maximizing throughput capacity or minimizing the communication latency in multihop wireless networks under the physical interference model. We build a unified algorithmic framework and develop approximation algorithms for link scheduling with or without power control.**

***Index Terms*—Link scheduling, physical interference model, maximum (concurrent) multiflow, approximation algorithm, maximum independent set.**

I. Introduction

Link scheduling is a fundamental problem in multihop wireless networks because the capacities of the communication links in multihop wireless networks, rather than being fixed, vary with the underlying link schedule subject to the wireless interference constraint. Precisely, we model a multihop wireless network by a triple $(V, A, \mathcal{I})$, where V is the set of networking nodes, A is the set of direct communication links among V, and $\mathcal{I}$ is the collection of sets of links in A which can transmit successfully at the same time. Each set in $\mathcal{I}$ is referred to as an *independent set*. A *(fractional) link schedule* is a set

$$S = \{(I_j, \ell_j) \in \mathcal{I} \times \mathbb{R}_+ : 1 \leq j \leq k\}.$$

A schedule S represented above is split into k time intervals of length $\ell_1, \ell_2, \cdots, \ell_k$ respectively. In the j-th time interval, all the links in I_j are scheduled to transmit. The value $\sum_{j=1}^{k} \ell_j$ is referred to as the length (or latency) of S, $|S|$ is called the size of S, and the function $c_S \in \mathbb{R}_+^A$ given by

$$c_S(e) = \sum_{j=1}^{k} \ell_j |I_j \cap \{e\}|, \forall e \in A$$

is called the link load function supported by S. The value $c_S(e)$ is the total transmission time of the link e throughout the duration of the schedule S. If the length of S is at most one, the function c_S gives rise to a link capacity function.

The optimization of wireless link scheduling has several variants. Link scheduling for maximizing throughput capacity have the following two variants:

- **Maximum Multiflow** (**MMF**): Given a multihop wireless network $(V, A, \mathcal{I})$ and a set of end-to-end communication requests specified by source-destination pairs, find a link schedule S of length at most one such that the maximum multiflow subject to the capacity function c_S is maximized.
- **Maximum Concurrent Multiflow** (**MCMF**): Given a multihop wireless network $(V, A, \mathcal{I})$ and a set of end-to-end communication requests specified by source-destination pairs together with their demands, find a link schedule S of length at most one such that the maximum concurrent multiflow subject to the capacity function c_S is maximized.

Link scheduling for minimizing latency is the following optimization problem:

- **Shortest Fractional Link Schedule** (**SFLS**): Given a multihop wireless network $(V, A, \mathcal{I})$ and a link demand function $d \in \mathbb{R}_+^A$, find a link schedule S of shortest length such that $c_S = d$.

The following two problems are closely related to link scheduling:

- **Maximum Independent Set of Links** (**MISL**): Given a multihop wireless network $(V, A, \mathcal{I})$ and a subset $A' \subseteq A$, find a set $I \in \mathcal{I}$ contained in A' with maximum size.
- **Maximum Weighted Independent Set of Links** (**MWISL**): Given a multihop wireless network $(V, A, \mathcal{I})$ and a link weight (or demand) function $d \in \mathbb{R}_+^A$, find a set $I \in \mathcal{I}$ with maximum total weight $d(I) = \sum_{a \in I} d(a)$.

The majority of algorithmic works on link scheduling in multihop wireless networks assume binary interference models such as the 802.11 interference model and the protocol interference model. Under a binary interference model, a set of links are conflict-free (i.e., belong to $\mathcal{I}$) if they are pairwise conflict-free. As a result, link scheduling under the binary interference model can employ the classic graph-theoretical tools such as graph coloring for algorithm design and analysis. Two widely adopted binary interference models are the 802.11 interference model and the protocol interference model. Under either of these two interference models, the networking nodes in V are typically assumed to lie in a planar region. Each node is associated with a circular communication range and a circular interference range centered at this node. Under the 802.11 interference model, the transmission from a node u to a node v succeeds if u and v are within each other's communication range, and neither of u and v is within the interference range of any other node transmitting or receiving at the same time. Under the protocol interference model, the transmission from a node u to a node v succeeds if v is within the communication range of u but is outside the interference range of any other node transmitting at the same time. But the binary interference model has to put a conservation and sometimes severe restrictions on interference ranges for practical applicability of the link schedules. The inefficiency of binary protocols compared to the physical model is well documented and has been shown theoretically as well as experimentally [13], [29], [32].

The physical interference model is more realistic and accurate than the binary interference model. Under the physical interference model, when a node u transmits a signal at power p, the power of this signal captured by another node v is $p \cdot \eta \|uv\|^{-\kappa}$, where $\|uv\|$ is the Euclidean distance between u and v, κ is *path-loss exponent* (a constant between 2 and 5 depending on the wireless environment), and η is the *reference loss factor*. The value $\eta \|uv\|^{-\kappa}$ is referred to as the path-loss factor of the pair (u, v). The signal quality perceived by a receiver is measured by the *signal to interference and noise ratio* (*SINR*), which is the quotient between the power of the wanted signal and the total power of unwanted signals and the ambient–both internal and external–noise. In order to correctly interpret the wanted signal, the SINR must be no less than certain threshold σ. There are two variants of the physical interference model: with or without the power control. In the variant without power control, the power assignment $p : V \times V \to \mathbb{R}_+$ is pre-specified. Typically, the pre-specified power assignment is *monotone* (i.e., for any two pairs (u, v) and (u', v') with $\|uv\| \leq \|u'v'\|$, $p(u, v) \leq p(u', v')$) and *sublinear* (i.e., for any two pairs (u, v) and (u', v') with $\|uv\| \leq \|u'v'\|$, $p(u, v) / \|uv\|^\kappa \geq p(u', v') / \|u'v'\|^\kappa$). The set A consists of all pairs (u, v) satisfying that

$$\|uv\| \leq \left(\frac{\eta p(u, v)}{\sigma \xi} \right)^{1/\kappa}.$$

A set I of links are independent (i.e., $I \in \mathcal{I}$) if and only if all links in I transmit simultaneously with the power assignment p, the SINR of each link in I is above σ. In the variant with power control, all nodes can adjust their transmission power to any value in a give set P. Let $p_{\max}$ and $p_{\min}$ be the maximum and minimum of P respectively. Then, A consists of all pairs (u, v) satisfying that

$$\|uv\| \leq \left(\frac{\eta \max_{p \in P} p}{\sigma \xi} \right)^{1/\kappa}.$$

A set I of links are independent (i.e., $I \in \mathcal{I}$) if and only if there exists a transmission power assignment to the links in I with values in P satisfying that when all links in I transmit simultaneously, the SINR of each link in I is above σ. In either variant, the *independence number* α of $\mathcal{I}$ refers to the size of a largest independent set in $\mathcal{I}$, i.e., $\alpha = \max_{I \in \mathcal{I}} |I|$.

The link scheduling problem under physical interference model is notoriously hard to resolve. The non-locality and the additive nature of interference in the physical interference model renders traditional techniques based on graph coloring inapplicable. Because of such technical challenge, most of the approximation bounds obtained in the literature are either trivial or grow linearly with the number of links. In this paper, we conduct a full-scale algorithmic study of **MMF**, **MCMF**, **SFLS**, and **MWISL** in multihop wireless networks under physical interference model. For any fixed monotone and sublinear power assignment, we show that all of **MWISL**, **SFLS**, **MMF**, and **MCMF** have a polynomial $O(\ln \alpha)$-approximation algorithm. For power control with unlimited maximum transmission power, we show that all of **MWISL**, **SFLS**, **MMF**, and **MCMF** also have a polynomial $O(\ln \alpha)$-approximation algorithm. For power control with P be the bounded set of possible values of transmission power of all nodes, our achieved approximation bounds involve the *power diversity* β of P, which is the smallest value k such that there exists a partition of P into k subsets in each of which any two elements differ by a factor of at most two. Note that

$$\beta \leq 1 + \log \frac{\max_{p \in P} p}{\min_{p \in P} p}.$$

We show that all of **MWISL**, **SFLS**, **MMF**, and **MCMF** have a polynomial $O(\beta \ln \alpha)$-approximation algorithm. In particular, in practical networks with constant power diversity β, all of them have a polynomial $O(\ln \alpha)$-approximation algorithm.

The remaining of this paper is organized as follows. In Section II, we review the literature on link scheduling in multihop wireless networks. In Section III, we build a unified framework of developing approximation algorithms for **MWISL**, **SFLS**, **MMF** and **MCMF** from an approximation algorithm for **MISL** in any multihop wireless network under any interference model. In Section IV, we present our approximation algorithms for **MISL**, **MWISL**, **SFLS**, **MMF**, and **MCMF** in multihop wireless networks under the physical interference model. In Section V, we summarize this paper and describe some open problems for further studies.

II. Related Works

There are a vast quantity of literature on network capacity of random wireless networks formed by a uniform or Poisson planar point process, under both protocol interference model and physical interference model. The asymptotics of network capacity in static wireless networks were studied in [1], [10], [14], [15], [23], [25], [28], [35], and the asymptotics of network capacity in mobile wireless networks were studied in [4], [14], [27], [34], [36]. However, as these results were largely non-algorithmic and restricted to networks with a well-behaving topology and traffic pattern such as uniform distribution, they do not directly help in understanding the capacity of arbitrary networks.

Link scheduling under the binary interference model is now fairly well understood [2], [5], [19], [21], [22], [24], [37], [40]. On the other hand, only some preliminary results on link scheduling under the physical interference model have been obtained very recently by a few research works. There are two variants of the link scheduling under the physical interference. In link scheduling with power control, the power assignment is part of the solution. In link scheduling with fixed power assignment, the power assignment is pre-specified. Typically, the pre-specified power assignment is *oblivious*, in other words, the transmission power of the sender of each link depends only on the path-loss factor of the link. There are three common oblivious power assignments. In the uniform power assignment, all senders of the links have the same transmission power; in the linear power assignment, the sender of a link transmits at a power proportional to the link's path-loss factor; in the square-root power assignment, the sender of a link transmits at a power proportional to the square-root of the link's path-loss factor. Note that all these three oblivious power assignments are monotone and sublinear. The advantage of oblivious power assignments is their simplicity which allows for simple implementation. However, for any oblivious power assignment there exists an instance of m links which are independent with power control but requires $\Omega(m)$ time-slots with this power assignment [8], [31].

The link scheduling problems which have been studied are either the problem **MISL** which seeks a maximum number of independent links of a given set of links, or the problem **Shortest Link Schedule** (**SLS**) which seeks a partition of a given set of links into the fewest independent sets. With uniform power assignment, both **MISL** and **SLS** are NP-hard [11]. With power control, **MISL** is also NP-hard [3]. With uniform power assignment, Halldórsson and Wattenhofer [18] and Wan et al. [38] obtained independently constant-approximation algorithms for **MISL**, the latter of which having better approximation bound. With fixed power assignment which is sublinear and length-monotone, Halldórsson and Mitra [17] gave a constant-approximation algorithm for **MISL** very recently. Other weaker (or even false) results on **MISL** with fixed power assignment have been obtained in [3], [11], [12] and [41]. With power control, Wan et al. [39] obtained an $O(\beta)$-approximation algorithm, where β is the power diversity. With power control but *without* noise (which is equivalent to unlimited maximum transmission power), Kesselheim [20] developed a constant-approximation algorithm for **MISL** very recently. When the physical interference model is adopted, particular attention should be paid to the assumption on the ambient noise, which is one of the major technical obstacle in achieving guaranteed approximation bound. The absence of noise effectively results in the *single-hop* wireless network in which every pair of nodes can directly with each other. Hence, any approximation bound without noise can only apply to single-hop wireless networks, but not to general multihop wireless networks. Other weaker results on **MISL** with power control have been obtained in [9] and [16]. These constant-approximation for **MISL** lead immediately to *logarithmic* approximation for **SLS** with the corresponding power assignments. Recently, Halldórsson and Wattenhofer [18] claimed a constant-approximation for **SLS** with uniform power assignment. But their algorithm (and its proof in Lemma 8 in [18]) is wrong, and the claim has been retracted by the authors very recently. Other weaker results on **SLS** have been obtained in [7] and [33].

Chafekar et al. [7] developed bi-criteria approximation algorithms for maximizing the throughput capacity. Due to the technical challenge caused by the noise, they compared the throughput rate guaranteed by their algorithms to the optimum rate possible if slightly lower power levels are used. With uniform transmission power or linear transmission power, the approximation bound is $O(\log \Lambda)$, where Λ is the ratio between the longest link length and the shortest link length; with power control, the approximation bound is $O(\log \Lambda \cdot \log \Gamma)$, where Γ is the ratio between the highest transmission power and the lowest transmission power. Note that the bound $O(\log \Lambda)$ may grow linearly with the number of links in general. In addition, the above approximation bounds are not the conventional ones as they are not compared to optima of the original instances, but to the smaller-valued optima of some restricted instances.

III. General Algorithmic Reductions in Arbitrary Wireless Networks

In this section, we build a unified framework of developing approximation algorithms for **MWISL**, **SFLS**, **MMF** and **MCMF** from an approximation algorithm for **MISL**. This framework is applicable to any multihop wireless network under *any* interference model. Throughout this section, a multihop wireless network is represented by a triple $(V, A, \mathcal{I})$ as described in Section I, and the independence number of $\mathcal{I}$ is denoted by α. Our algorithmic framework is described in the next theorem.

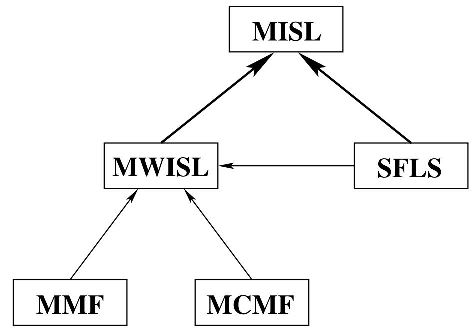

Fig. 1. Polynomial algorithmic reductions: the two reductions to **MISL** expand the approximation ratios by a logarithmic factor, and the three reductions to **MWISL** are approximation-preserving.

Theorem 1: Suppose that **MISL** has a polynomial μ-approximation algorithm. Then,

1) **MWISL**, **MMF** and **MCMF** all have a polynomial $e(1 + \ln \alpha)\mu$-approximation algorithm, where $e \approx 2.718$ is the natural base.
2) **SFLS** has a polynomial $(1 + \mu \ln \alpha)$-approximation algorithm.

The proof of Theorem 1 involves the polynomial algorithmic reductions illustrated in Figure 1. The three reductions to **MWISL** from **SFLS**, **MMF** and **MCMF** respectively are approximation-preserving, while the two reductions to **MISL** from **MWISL** and **SFLS** respectively increases the approximation ratio by a logarithmic factor. The three reductions to **MWISL** use the ellipsoid method for linear programming together with an approximate separation oracle. This same reduction is a generalization to Theorem 4.1 in [37], which assumes either 802.11 interference model or the protocol interference model. However, the same algorithm and its analysis can be extended to an arbitrary interference model. The detail of the reduction and its proof can be found in [37] and is omitted here. The detailed reductions to **MISL** from **MWISL** and **SFLS** respectively are given below. Let $\mathcal{A}$ be a polynomial μ-approximation algorithm for **MISL**. We will present a polynomial $(1 + e\mu \ln \alpha)$-approximation algorithm for **MWISL** and a polynomial $(1 + \mu \ln \alpha)$-approximation algorithm for **SFLS**, both of which employ the algorithm $\mathcal{A}$. Theorem 1 then follows immediately from these algorithmic reductions.

We first describe our polynomial $(1+e\mu\ln\alpha)$-approximation algorithm for **MWISL**. Let $d \in \mathbb{R}_+^A$ be the given link demand function. We sort the links in A in the decreasing order of the demands: $a_1, a_2, \cdots, a_m$. Then, we greedily partition A into subsets $A_1, A_2, \cdots, A_k$ following this order satisfying that in each that in each subset A_j with $1 \leq j \leq k$ the demand of the heaviest link is at most e times the demand of the lightest link. After that, we apply the algorithm $\mathcal{A}$ on each A_j with $1 \leq j \leq k$ to select an independent set I_j of links in A_j. Finally, we output the set I which is the heaviest one among these k independent sets and the singleton independent set $\{a_1\}$. Clearly, the algorithm runs in polynomial time. Below, we show that I is a $(1+e\mu\ln\alpha)$-approximate solution.

Let I^* be a maximum weighted independent set in $\mathcal{I}$. We claim that for each $1 \leq j \leq k$,

$$d\left(I^* \cap A_j\right) \leq \min\left\{e\mu, \frac{|I^* \cap A_j|}{e^{j-1}}\right\} d\left(I\right).$$

Indeed,

$$\begin{aligned} d\left(I^* \cap A_j\right) &\leq |I^* \cap A_j| \cdot \max_{a \in A_j} d\left(a\right) \\ &\leq \mu \left|I_j\right| \cdot e \min_{a \in A_j} d\left(a\right) \\ &\leq e\mu \left|I_j\right| \min_{a \in I_j} d\left(a\right) \\ &\leq e\mu \cdot d\left(I_j\right) \\ &\leq e\mu \cdot d\left(I\right). \end{aligned}$$

It's easy to prove by induction that for each $a \in A_j$, $d\left(a\right) \leq d\left(a_1\right)/e^{j-1}$. Therefore,

$$\begin{aligned} d\left(I^* \cap A_j\right) &\leq |I^* \cap A_j| \cdot \max_{a \in A_j} d\left(a\right) \\ &\leq |I^* \cap A_j| \frac{d\left(a_1\right)}{e^{j-1}} \\ &\leq \frac{|I^* \cap A_j|}{e^{j-1}} d\left(I\right). \end{aligned}$$

So, our claim holds.

Now, we show that

$$d\left(I^*\right) \leq e\left(1+\mu\ln\alpha\right) d\left(I\right).$$

If $\alpha \leq e$, then

$$d\left(I^*\right) = \alpha \cdot d\left(I\right) \leq e \cdot d\left(I\right).$$

So, we assume that $\alpha > e$. By the previous claim, we have

$$\begin{aligned} &\sum_{j=1}^{\lceil\ln\alpha\rceil-1} d\left(I^* \cap A_j\right) \\ &\leq \left(\lceil\ln\alpha\rceil - 1\right) e\mu \cdot d\left(I\right) \\ &\leq \left(e\mu\ln\alpha\right) d\left(I\right) \end{aligned}$$

and

$$\begin{aligned} &\sum_{j=\lceil\ln\alpha\rceil}^{k} d\left(I^* \cap A_j\right) \\ &\leq d\left(I\right) \sum_{j=\lceil\ln\alpha\rceil}^{k} \frac{|I^* \cap A_j|}{e^{j-1}} \\ &\leq d\left(I\right) \frac{1}{e^{\lceil\ln\alpha\rceil}} \sum_{j=\lceil\ln\alpha\rceil+1}^{k} |I^* \cap A_j| \\ &= \frac{\alpha}{e^{\lceil\ln\alpha\rceil}} \cdot d\left(I\right) \\ &\leq e \cdot d\left(I\right). \end{aligned}$$

Hence,

$$d\left(I^*\right) = \sum_{j=1}^{k} d\left(I^* \cap A_j\right) \leq e\left(1+\mu\ln\alpha\right) d\left(I\right).$$

Therefore, I is a $(1+e\mu\ln\alpha)$-approximate solution.

Next, we describe our polynomial $(1+\mu\ln\alpha)$-approximation algorithm for **SFLS**. Let $d \in \mathbb{R}_+^A$ be the given link demand function. We construct a fractional link schedule S of d in the following iterative manner. At each iteration, let A' be the subset of links e in A with $d\left(e\right) > 0$. We apply the algorithm $\mathcal{A}$ on A' to select an independent set I of links in A'. Let $\ell = \min_{v \in I} d\left(v\right)$, and add $\left(I, \ell\right)$ to S. For each link a in I, replace $d\left(a\right)$ by $d\left(a\right) - \ell$. Repeat this iteration if $d \neq 0$. Since in each iteration the number of links a with $d\left(a\right) > 0$ strictly decreases, the number of iterations, or the size of the schedule S, is bounded by $|A|$. Below, we show that S is a $(1+\mu\ln\alpha)$-approximate solution.

Suppose that the algorithm runs in k iterations. For each $1 \leq j \leq k$, let $d_j \in \mathbb{R}_+^A$ be the residue demand at the beginning of the j-th iteration, A_j be the set of links in A with positive reside demands, $\left(I_j, \ell_j\right) \in \mathcal{I} \times \mathbb{R}_+$ be the pair selected in the j-th iteration. Let $d_{k+1} \in \mathbb{R}_+^A$ be the zero-demand function. Then,

$$d_1\left(A\right) > d_2\left(A\right) > \cdots > d_{k+1}\left(A\right) = 0,$$

and for each $1 \leq j \leq k$,

$$\ell_j \left|I_j\right| = d_j\left(A\right) - d_{j+1}\left(A\right).$$

For each $1 \leq j \leq k$, let α_j be the size of a maximum independent set of links in A_j, and opt_j be the minimum latency of d_j. Then,

$$opt_j \leq d_j(A) \leq \alpha_j \cdot opt_j.$$

and

$$opt_1 \geq opt_2 \geq \cdots \geq opt_k.$$

Thus,

$$d_j(A) \leq \alpha_j \cdot opt_j \leq \mu |I_j| \cdot opt_1,$$

which implies

$$|I_j| \geq \frac{1}{\mu} \frac{d_j(A)}{opt_1}.$$

Since

$$d_1(A) > d_2(A) > \cdots > d_{k+1}(A) = 0,$$

and $opt_1 \leq d_1(A)$, there exists a unique index t satisfying $d_t(A) \geq opt_1 > d_t(A)$. Thus,

$$\begin{aligned}
&\sum_{j=1}^{t-1} \ell_j + \frac{d_t(A) - opt_1}{d_t(A) - d_{t+1}(A)} \ell_t \\
&= \sum_{j=1}^{t-1} \frac{d_j(A) - d_{j+1}(A)}{|I_j|} + \frac{d_t(A) - opt_1}{|I_t|} \\
&\leq \sum_{j=1}^{t-1} \frac{d_j(A) - d_{j+1}(A)}{\frac{1}{\mu}\frac{d_j(A)}{opt_1}} + \frac{d_t(A) - opt_1}{\frac{1}{\mu}\frac{d_t(A)}{opt_1}} \\
&= \mu \cdot opt_1 \left(\sum_{j=1}^{t-1} \frac{d_j(A) - d_{j+1}(A)}{d_j(A)} + \frac{d_t(A) - opt_1}{d_t(A)} \right) \\
&\leq \mu \cdot opt_1 \int_{opt_1}^{d_1(A)} \frac{1}{x} dx \\
&= \mu \cdot opt_1 \ln \frac{d_1(A)}{opt_1} \\
&\leq \mu \cdot opt_1 \ln \alpha_1 \\
&\leq \mu \cdot opt_1 \ln \alpha.
\end{aligned}$$

In addition, we have

$$\begin{aligned}
&\frac{opt_1 - d_{t+1}(A)}{d_t(A) - d_{t+1}(A)} \ell_t + \sum_{j=t+1}^{k} \ell_j \\
&= \frac{opt_1 - d_{t+1}(A)}{|I_t|} + \sum_{j=t+1}^{k} \frac{d_j(A) - d_{j+1}(A)}{|I_j|} \\
&\leq opt_1 - d_{t+1}(A) + \sum_{j=t+1}^{k} (d_j(A) - d_{j+1}(A)) \\
&= opt_1 - d_{k+1}(A) \\
&= opt_1.
\end{aligned}$$

Hence,

$$\begin{aligned}
&\sum_{j=1}^{k} \ell_j \\
&= \sum_{j=1}^{t-1} \ell_j + \frac{d_t(A) - opt_1}{d_t(A) - d_{t+1}(A)} \ell_t \\
&\quad + \frac{opt_1 - d_{t+1}(A)}{d_t(A) - d_{t+1}(A)} \ell_t + \sum_{j=t+1}^{k} \ell_j \\
&\leq \mu \cdot opt_1 \ln \alpha + opt_1 \\
&= (1 + \mu \ln \alpha) opt_1.
\end{aligned}$$

Thus, S is a $(1 + \mu \ln \alpha)$-approximate solution.

We remark that if the link demand function $d \in \mathbb{R}_+^A$ is integral, then the fractional link schedule produced by our polynomial $(1 + \mu \ln \alpha)$-approximation algorithm for **SFLS** is also integral. In particular, our polynomial $(1 + \mu \ln \alpha)$-approximation algorithm for **SFLS** is also a our polynomial $(1 + \mu \ln \alpha)$-approximation algorithm for **SLS**. We also would like to point out the polynomial approximation-preserving reduction from **SFLS** to **MWISL** together with the first part of Theorem 1 can lead to a polynomial $e(1 + \ln \alpha)\mu$-approximation algorithm for **SFLS**. However, the approximation bound $e(1 + \ln \alpha)\mu$ is weaker than the approximation bound $1 + \mu \ln \alpha$ given in the second part of Theorem 1. In addition, our direct polynomial reduction from **SFLS** to **MISL** is much simpler than the chained reductions from **SFLS** to **MWISL** and from **MWISL** to **MISL**. A natural question is whether there exists any approximation-preserving reduction from **MWISL**, **MMF**, or **MCMF** to **SFLS**. This question remains open by now.

IV. Link Scheduling under Physical Interference Model

In this section, we apply Theorem 1 to devise polynomial approximation algorithms for **MMF**, **MCMF**, **SFLS**, and **MWISL** under the physical interference model. Let $(V, A, \mathcal{I})$ be the triple representation of a multihop wireless network under the physical interference model as described in Section I. We denote by α the independence number of $\mathcal{I}$.

For a fixed monotone and sublinear power assignment, a constant-approximation algorithm for **MISL** was given in [17]. Let $\mathcal{A}_1$ denote such algorithm and μ_1 be its approximation radio. By Theorem 1, we immediately have the following approximation results.

Theorem 2: For any fixed monotone and sublinear power assignment,

1) **SFLS** has a polynomial $(1 + \mu_1 \ln \alpha)$-approximation algorithm;
2) **MWISL**, **MMF**, and **MCMF** all admit a polynomial $e(1 + \ln \alpha_1)\mu$-approximation algorithm.

Next, we assume power control with P be the bounded set of possible values of transmission power of all nodes. We denote by β the power diversity of P. For uniform power assignment (i.e., P is a singleton), a constant-approximation algorithm for **MISL** was given in [38]. Let $\mathcal{A}_2$ denote such algorithm and μ_2 be its approximation radio. We simply adopt the uniform maximum power assignment in which all nodes transmit at the maximum power in P, and apply $\mathcal{A}_2$ to find an independent set I under this uniform maximum power assignment. It was shown in [39] that the output I is a $16\beta\mu_2$-approximate solution for **MISL** with power control. So, by Theorem 1 we have the following approximation results.

Theorem 3: For power control with bounded power diversity β,

1) **SFLS** has a polynomial $(1 + 16\beta\mu_2 \ln \alpha)$-approximation algorithm;
2) **MWISL**, **MMF**, and **MCMF** all admit a polynomial $16e\beta\mu_2(1 + \ln \alpha)$-approximation algorithm.

Finally, we assume power control with unlimited maximum transmission power. In this case, a constant-approximation algorithm for **MISL** was given in [20]. Let $\mathcal{A}_2$ denote such algorithm and μ_3 be its approximation radio. Then, Theorem 1 imply the following approximation results.

Theorem 4: For power control with unlimited maximum transmission power,

1) **SFLS** has a polynomial $(1 + \mu_3 \ln \alpha)$-approximation algorithm;
2) **MWISL**, **MMF**, and **MCMF** all admit a polynomial $e(1 + \ln \alpha)\mu_3$-approximation algorithm.

V. Conclusion

In this paper, we have built a unified framework of developing approximation algorithms for **MWISL**, **SFLS**, **MMF** and **MCMF** from an approximation algorithm for **MISL** in any multihop wireless network under any interference model. Following such framework, we showed for any fixed monotone and sublinear power assignment or for power control with unlimited maximum transmission power, all of **MWISL**, **SFLS**, **MMF**, and **MCMF** have a polynomial $O(\ln \alpha)$-approximation algorithm, where α is the independence number of $\mathcal{I}$; For power control with bounded power diversity β, all of **MWISL**, **SFLS**, **MMF**, and **MCMF** have a polynomial $O(\beta \ln \alpha)$-approximation algorithm. In particular, in practical networks with constant power diversity β, all of them have a polynomial $O(\ln \alpha)$-approximation algorithm.

There are still many interesting and challenging unresolved research issues on the link scheduling subject to the physical interference. First, for power control with bounded set of transmission power, it remains open whether **MISL** admits a polynomial constant-approximation algorithm. A positive answer to this open problem will imply the existence of polynomial $O(\ln \alpha)$-approximation algorithms for **SFLS**, **MMF** and **MCMF**. Second, it is unknown whether **MWISL** admits a polynomial constant-approximation algorithm even when all nodes have fixed uniform transmission power. Third, it is also unknown whether the same approximation results can be achieved in multi-channel multi-radio wireless networks under the physical interference model.

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