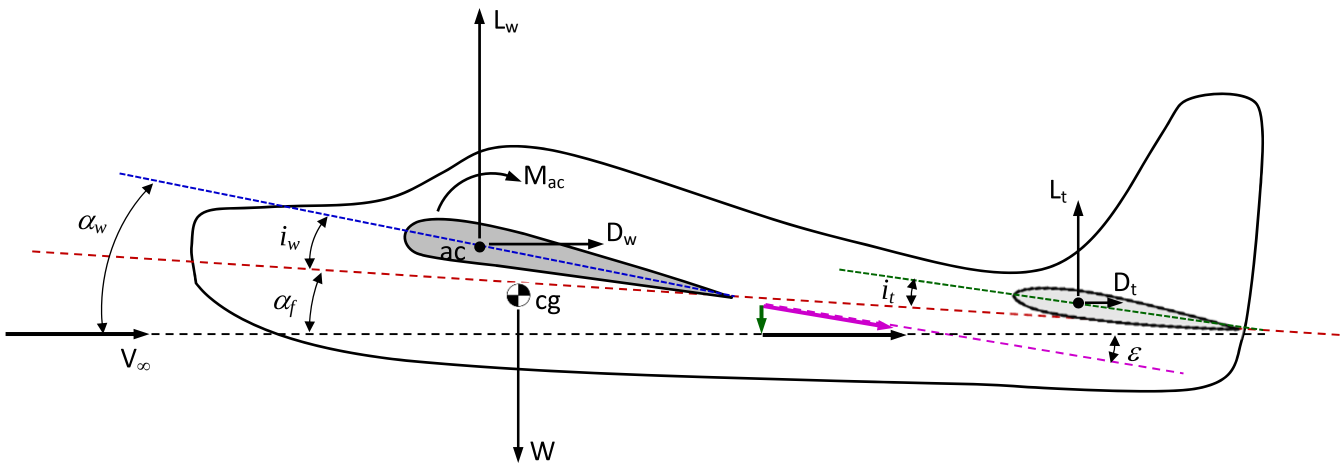
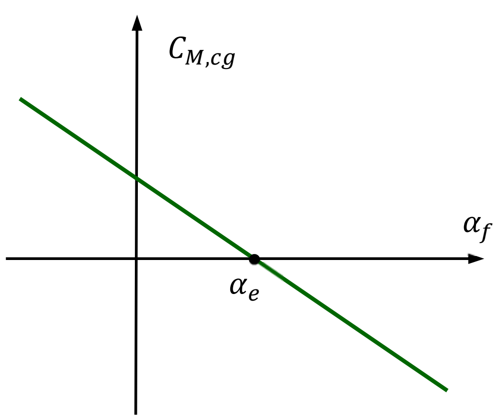
STATIC PITCH STABILITY of an AIRPLANE:



Longitudinal (pitch) stability requires a moment about cg ($C_{M,cg}$) vs. angle of attack (α_f) curve as in the following graph:



There is a certain equilibrium AoA (α_e , denoted by solid circle in the graph) at which the pitching moment about cg becomes zero. This corresponds to level flight conditions without any pitching about the cg point. Negative slope of the moment curve provides the static stability such that if the AoA increases (say, due to a momentary gust), a negative pitching moment brings the airplane AoA back to the equilibrium point.

Note that if the airplane is designed for a level cruising flight along the fuselage axis (red dotted line), then the AoA of airplane becomes zero. Therefore, in such a design, equilibrium point has to be achieved at zero AoA in the moment curve (i.e., $\alpha_e = 0$).

MAIN WING: From the above sketch we can write the pitching moment of the main wing about cg as follows:

$$M_{cg_w} = M_{ac_w} + L_w cos(\alpha_w)(\bar{x}_{cg} - \bar{x}_{ac})c + D_w sin(\alpha_w)(\bar{x}_{cg} - \bar{x}_{ac})c$$

$$-L_w sin(\alpha_w) z_w c + D_w cos(\alpha_w) z_w c$$

where $\alpha_w = (\alpha_f + i_w)$ and z_w is distance of cg point from wing's chord line. For the normal flight range of a conventional airplane, α_w is small; hence we can approximate that

$$cos(\alpha_w) \cong 1 \text{ and } sin(\alpha_w) \cong \alpha_w$$

By using this approximation and dividing the both sides of above moment equation by $(q_\infty Sc)$ we obtain:

$$C_{M,cg_w} = C_{M,ac_w} + (C_{L,w} + C_{D,w}\alpha_w)(\bar{x}_{cg} - \bar{x}_{ac}) - (C_{L,w}\alpha_w - C_{D,w})z_w$$

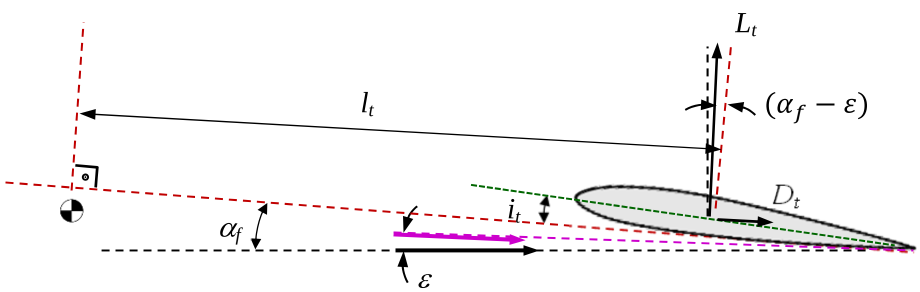
For most airplanes cg is located close to the chord line ; hence z is usually small ($z \cong 0$) and will be neglected. Furthermore, α_w (in radians) is usually much less than unity, and $C_{D,w}$ is less than $C_{L,w}$; hence the product $C_{D,w}\alpha_w$ is small in comparison to $C_{L,w}$. With these assumptions, the moment about cg point can be reduced to:

$$C_{M,cg_w} = C_{M,ac_w} + C_{L,w}(\bar{x}_{cg} - \bar{x}_{ac})$$

or in terms of angle of attack:

$$C_{M,cg_w} = C_{M,ac_w} + a_w(\alpha_w - \alpha_{w,0l})(\bar{x}_{cg} - \bar{x}_{ac})$$

<u>HORIZONTAL TAIL:</u> Following a similar approach, we can compute the pitching moment due to the horizontal tail. However, before doing so, we must consider the effect of downwash on the tail. Existence of downwash decreases the angle of streamlines impingent on the horizontal tail by an amount, ε, which is called the downwash angle (see the figure below). The downwash angle (ε) is larger immediately after the main wing but falls off pretty quickly and approaches the value at infinity within 3-4 chord lengths (downwash angle will be larger the closer the tail gets to the wing, NACA TR648).



The downwash angle at infinity, usually used as first estimation of the AoA at the tail, is:

$$\varepsilon = \frac{2C_L}{\pi AR} = \frac{2a_w(\alpha_w - \alpha_{w,0l})}{\pi AR} = \frac{2a_w(\alpha_f + i_w - \alpha_{w,0l})}{\pi AR}$$

Note that the downwash angle at the wing quarter-chord is half the value at infinity, i.e.,

$$\alpha_i = \frac{C_L}{\pi AR}$$

Now, we can compute the horizontal tail's pitching moment about airplane's cg point as follows:

$$M_{cg_t} = -l_t\left[L_t\cos\left(\alpha_f - \varepsilon\right) + D_t\sin\left(\alpha_f - \varepsilon\right)\right] - z_t\left[L_t\sin\left(\alpha_f - \varepsilon\right) - D_t\cos\left(\alpha_f - \varepsilon\right)\right] + M_{ac_t}$$

The first term on the right side, $l_t L_t\cos\left(\alpha_f - \varepsilon\right)$, is by far the largest in magnitude. In fact, for conventional airplanes, the following simplifications are reasonable:

1) $z_t \ll l_t$;
2) $D_t \ll L_t$;
3) $\sin\left(\alpha_f - \varepsilon\right) \cong 0$;
4) $\cos(\alpha - \varepsilon) \cong 1$;
5) and M_{ac_t} is small in magnitude.

With the preceding approximations, which are based on experience, contribution to the pitching moment due to horizontal tail dramatically simplifies to:

$$M_{cg_t} = -l_t L_t = -l_t C_{L,t} q_\infty S_t$$

Dividing both sides by $(q_\infty Sc)$ yields

$$C_{M,cg_t} = -\frac{l_t S_t}{Sc} C_{L,t} = -V_H C_{L,t}$$

This simple relation gives the total contribution of the tail to the moments about the airplane's center of gravity. Here V_H is called *tail volume ratio*. The student must note that the effect of downwash on the tail moment is hidden within the tail's lift coefficient. To see it better, it is useful to write down the angle of attack of the tail:

$$\alpha_t = \alpha_f + i_t - \varepsilon$$

Hence;

$$C_{M,cg_t} = -V_H C_{L,t} = -V_H a_t \alpha_t = -V_H a_t(\alpha_f + i_t - \varepsilon)$$

$$C_{M,cg_t} = -V_H a_t\left\{\alpha_f + i_t - \frac{2a_w\left(\alpha_w - \alpha_{w,0l}\right)}{\pi AR}\right\}$$

In above expression, it is assumed that horizontal tail has a symmetric airfoil.

<u>TOTAL PITCHING MOMENT about CG:</u> Combining the contributions from the main wing and horizontal tail to the pitching moment about the cg point yields:

$$C_{M,cg} = C_{M,ac_w} + C_{L,w}\left(\bar{x}_{cg} - \bar{x}_{ac}\right) - V_H C_{L,t}$$

and in terms of angle of attack:

$$C_{M,cg} = C_{M,ac_w} + a_w\left(\alpha_f + i_w - \alpha_{w,0l}\right)\left(\bar{x}_{cg} - \bar{x}_{ac}\right) - V_H a_t \left\{\alpha_f + i_t - \frac{2a_w\left(\alpha_f + i_w - \alpha_{w,0l}\right)}{\pi AR}\right\}$$

We have seen that the criteria necessary for longitudinal balance and static stability is that (i) $C_{M,cg} = 0$ at cruise attitude and (ii) $\partial C_{M,cg}/\partial \alpha_f$ must be negative, both conditions with the implicit assumption that α_e falls within the practical flight range of angle of attack; i.e., that is moment coefficient curve must be similar to that sketched at the beginning of this section. The slope of the moment coefficient curve is obtained by differentiating with respect to α_f

$$\frac{\partial C_{M,cg}}{\partial \alpha_f} = a_w\left(\bar{x}_{cg} - \bar{x}_{ac}\right) - V_H a_t \left\{1 - \frac{2a_w}{\pi AR}\right\}$$

This equation clearly shows the powerful influence of the location $\bar{x}_{cg}$ of the center of gravity and the tail volume ratio V_H in determining pitch stability. It also allows us to establish a certain philosophy in the design of an airplane, which will be discussed next.

<u>NEUTRAL POINT:</u> Consider the situation where the location $\bar{x}_{cg}$ of the center of gravity is allowed to move with everything else remaining fixed. In fact, the last equation indicates that static stability is a strong function of $\bar{x}_{cg}$. Indeed, the value of $\partial C_{M,cg}/\partial \alpha_f$ can always be made negative by properly locating the center of gravity. There is one specific location of the CG such that $\partial C_{M,cg}/\partial \alpha_f = 0$. The value $\bar{x}_{cg}$ when this condition holds is defined as the *neutral point*, denoted by $\bar{x}_{np}$. When $\bar{x}_{cg} = \bar{x}_{np}$, the slope of the moment coefficient curve becomes zero, and moving the $\bar{x}_{cg}$ further aft of the airplane beyond neutral point results in an unstable airplane, i.e., $\partial C_{M,cg}/\partial \alpha_f > 0$.

The location of the neutral point is readily obtained by setting $\bar{x}_{cg} = \bar{x}_{np}$, and $\partial C_{M,cg}/\partial \alpha_f = 0$, as follows:

$$\frac{\partial C_{M,cg}}{\partial \alpha_f} = 0 = a_w\left(\bar{x}_{np} - \bar{x}_{ac}\right) - V_H a_t \left\{1 - \frac{2a_w}{\pi AR}\right\}$$

Solving this equation for $\bar{x}_{np}$ yields:

$$\bar{x}_{np} = \bar{x}_{ac} + V_H \frac{a_t}{a_w}\left\{1 - \frac{2a_w}{\pi AR}\right\}$$

The quantities on the right side are, for all practical purposes, established by the design configuration of the airplane. Thus, for a given airplane design, the neutral point is a *fixed quantity* (i.e., a point that is frozen somewhere on the airplane). It is quite independent of the actual location $\bar{x}_{cg}$ of the center of gravity. *For static pitch stability, the position of the CG must always be forward of the NP*. One must note that the horizontal stabilizer strongly influence the location the NP. *By proper selection of the tail parameters, principally V_H, the location of neutral point ($\bar{x}_{np}$) can be adjusted by the designer*.

Another simplified relation that relates the location of neutral point to tail volume ratio is often given by:

$$\bar{x}_{np} \cong \frac{1}{4} + \frac{1 + 2/AR}{1 + 2/AR_H}\left(1 - \frac{4}{2 + AR}\right)V_H$$

The student must note that this expression is based on the assumption that airfoils' lift slope is given by $dC_l/d\alpha = 2\pi$, which is not always correct particularly at low Reynolds numbers. Therefore, the previous relation must be used together with 3D wing lift slopes ($a = dC_L/d\alpha$) whenever they are available from experimental data or XFLR5 type computational codes.

<u>STATIC MARGIN:</u> Preceding discussion of the neutral point can be extended to derive an extremely useful measure of pitch stability, which is referred to as *static margin*. To this end, let's solve the above equation for $\bar{x}_{ac}$:

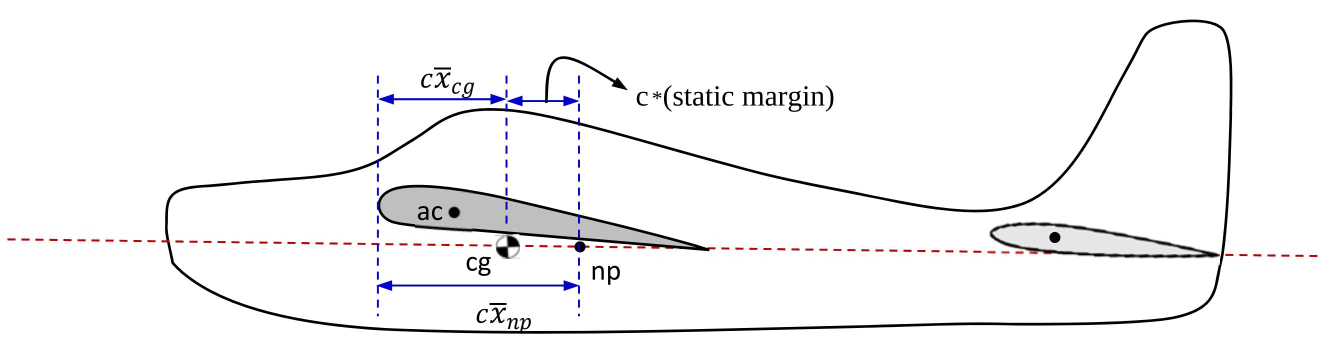
$$\bar{x}_{ac} = \bar{x}_{np} - V_H \frac{a_t}{a_w}\left\{1 - \frac{2a_w}{\pi AR}\right\}$$

and substitute it in $\partial C_{M,cg}/\partial \alpha_f$ relation:

$$\frac{\partial C_{M,cg}}{\partial \alpha_f} = a_w\left(\bar{x}_{cg} - \bar{x}_{np} + V_H \frac{a_t}{a_w}\left\{1 - \frac{2a_w}{\pi AR}\right\}\right) - V_H a_t\left\{1 - \frac{2a_w}{\pi AR}\right\}$$

which yields:

$$\frac{\partial C_{M,cg}}{\partial \alpha_f} = -a_w(\bar{x}_{np} - \bar{x}_{cg}) = -a_w \times Static\ Margin$$



The distance $(\bar{x}_{np} - \bar{x}_{cg})$ is defined as the *static margin*, and it is a direct measure of static pitch stability. For static stability, the static margin must be positive. Moreover, the larger the static margin, the more stable the airplane.

For conventional R/C airplane designs, the CG location range is usually between 28% and 33% from the leading edge of the main wing's MAC, which corresponds to about 5% to 15% ahead of the airplane's neutral point (NP), i.e., a static margin of about 5% to 15%. The CG location as described above is pretty close to the wing's AC because the lift due to horizontal stabilizer has only slight effect on conventional R/C models.

TAIL SIZING: For a well-behaved airplane, tail volume ratio for horizontal tail and vertical tail generally falls into the ranges given below:

$$V_H = \frac{l_t S_t}{Sc} = 0.30 - 0.60$$

for the horizontal tail, and

$$V_V = \frac{l_v S_v}{Sb} = 0.02 - 0.05$$

for the vertical tail. Note that the scaling factor in the denominator of tail volume ratio is composed of MAC length (c) for the main wing, while it is replaced with wing span (b) for the vertical tail.

DESIGN PROCESS to set the INCIDENCE ANGLES: The design process to set the wing incidence (i_w) and horizontal tail incidence (i_t) angles should be:

1. choose the cruise speed,
2. find the wing's lift coefficient $C_{L,w}$ required,
3. find the fuselage angle (α_f) for the least drag at cruising speed,
4. determine the wing incidence angle (i_w) that will provide $C_{L,w}$ required
5. determine CG location ($\bar{x}_{cg}$) for desired pitch stability (*tip: keeping CG close to AC of MAC results in minimal tail lift to achieve pitch stability and, hence, minimize the induced drag – trim drag – due to tail lift*),
6. determine the tail volume ratio (V_H) and, then, the tail size and location,
7. compute the tail incidence angle (i_t) to trim at desired speed (i.e., to provide $C_{L,t}$ required to establish pitch stability) accounting for the wing downwash angle at that $C_{L,w}$.